Junior High Number Sense – Special Topics

Competition in Spring 2021 and Spring 2022

This document contains information on some of the new tricks that will appear on the 2021 and 2022 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given explicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

Squaring "Hollow" Numbers [#20-40]

In this problem, students will be asked to find the square of three-digits numbers with a middle digit of 0. Some examples include 405^2, 702^2, and 309^2. The student is encouraged to work out an easy mental math trick for problems of this type. You might consider working several of these problems out and looking for the obvious pattern. Then, try to provide an algebraic proof that your conjecture is correct. Good luck!

Special Fraction Addition [#40-60]

Problems of this type will be in the form $\frac{k}{n} + \frac{k}{2n} + \frac{k}{4n}$, where k and n are whole numbers. To add these fractions together, you need to get a common denominator. The least common denominator of all three fractions is $4n$. This gives

$$\frac{k}{n} + \frac{k}{2n} + \frac{k}{4n} = \frac{k}{n} \cdot \frac{4}{4} + \frac{k}{2n} \cdot \frac{2}{2} + \frac{k}{4n} = \frac{4k}{4n} + \frac{2k}{4n} + \frac{k}{4n} = \frac{4k + 2k + k}{4n} = \boxed{\frac{7k}{4n}}$$

For example, in the problem $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, the value of k is 1 and the value of $4n = 8$. You can see that $n = 2$ from the first fraction, but you really just need the value of $4n$ since it is the denominator used in the answer. The answer is $\frac{7k}{4n} = \frac{7 \cdot 1}{8} = \frac{7}{8}$.

Another example, $4\frac{7}{9} + 5\frac{7}{18} + 6\frac{7}{36}$. At first glance, this does not look like this problem type, but the fractionals parts of the mixed numbers do form the pattern. Add these fractions using the trick: $\frac{7 \cdot 7}{36} = \frac{49}{36}$. Since this fraction is improper, you need to subtract 1 to make it proper and then carry this 1 over to the whole numbers. To subtract 1 from a fraction, subtract the numerator minus the denominator: $49 - 36 = 13 \quad \longrightarrow \quad \frac{13}{36}$

The sum of the whole numbers is $4 + 5 + 6 + 1_{\text{(carry)}} = 16$. The answer is $16\frac{13}{36}$.

Permutations [#60-80]

To understand permutations, you should first understand **factorials**. The symbol $n!$ stands for "n factorial." The value of $n!$ is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$. It is the product of every whole number from 1 up to n. For example, $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

The symbol for permutation is ${}_nP_r$ and it is defined to equal $\dfrac{n!}{(n-r)!}$. But there is a trick! Let's see an example:

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot\cdot 1} = 8\cdot 7\cdot 6 = 336$$

The trick is to notice that the numbers in purple are really the numbers that should be multiplied to get the answer. The numbers start at 8 and there are 3 numbers in the sequence. These are the numbers from the permutation ${}_8P_3$.

The permutation number is the number of ways r objects can be selected **in order** from a group of n total objects. The number of ways 3 runners can finish a race when 8 runners ran is the number found by ${}_8P_3$, because the order that the runners finish in matters. First place is better than second place, which is better than third place.

Equation of the Circle [#60-80]

The equation of the graph of the circle is $(x-h)^2 + (y-k)^2 = r^2$, where the center of the circle is located at the point (h, k) and the radius of the circle is r.

Example:
Find the y-coordinate of the center of the circle whose equation is $(x-3)^2 + (y+1)^2 = 9$.

Solution:
The equation of the circle gives the y-coordinate in the $(y-k)^2$ part. Notice that this is written with a minus sign. In the problem, we have a plus sign. Thus, you should think about the numbers given in the equation as the opposite of their actual coordinates. The y-coordinate is -1.

Example:
Find the radius of the circle whose equation is $x^2 + 8x + y^2 - 2y = 8$.

Solution:
The equation is not presented in the correct form to read off the radius. To change its form, we need to complete the square. We complete the square on each of the x's and y's. For the x's, take half of the 8 and then square that number: $\left[\frac{1}{2}(8)\right]^2 = 16$. For the y's, take half of the 2 (you can ignore the negative here), and then square that number: $\left[\frac{1}{2}(2)\right]^2 = 1$. Add 16 and 1 to the right side: $8 + 16 + 1 = 25$. This 25 is the square of the radius and so the radius is 5.

Other types of questions that can be asked here include finding the area or circumference of a circle from its equation, finding one coordinate of a point on a circle when the other coordinate has been given (with another condition), etc.

For practice questions, visit `www.academicmeet.com` and click on Resources.