# New Questions for Junior High Number Sense (2013 and 2014)

This document contains information on some of the new tricks that will be appearing on the 2013 and 2014 Number Sense tests.

The new tricks are sectioned in the order that they will appear on the test. Some of the new tricks are given implicitly. For the others, the student is encouraged to search for an easy mental math formula, procedure for working the problem, or information on the topic.

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MULTIPLICATION BY 13 [#20-40]

The trick for multiplying by 13 is similar to the multiplying by 12 trick. Instead of "doubling and adding back to the right", you will "triple and add back to the right." For example, to calculate $45 \times 13$, start by tripling the 5 to get 15. Write the 5 on the right side of the answer and carry the 1. Move to the left one digit to the 4. Triple the 4 to get 12 and then add the 5 on the right to get 17. Add the carry from before to get 18. Write the 8 to the left of the 5 and carry the 1. Finally, triple the 0 (on the left side of 45) and then add the 4 and the 1 from the carry. Write the 5 on the left. The product is 585.

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SPECIAL 3-DIGIT MULTIPLICATION [#40-60]

Problems in the form of $913 \times 413$ will appear in this section. The pattern to notice is that both numbers have three digits, the last two digits of both numbers are the same, and the hundred's places from the numbers add up to give you that two-digit number. Here, $9 + 4 = 13$. I will leave it to you to develop a trick to handle this product.

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SUM OF THE FIRST $n$ TRIANGULAR NUMBERS [#60-80]

Recall that the triangular numbers are the numbers 1, 3, 6, 10, 15, ..., where the $n$th number in the sequence is the sum $1 + 2 + 3 + \ldots + n$. A natural extension is to ask what happens when you add triangular numbers together.

The sum of the first $n$ triangular numbers is given by the formula

$$S = \frac{n(n+1)(n+2)}{6}.$$

For example, the sum of the first 5 triangular numbers is $1+3+6+10+15 = 35$. By the formula, the sum is $S = \frac{5(6)(7)}{6} = 35$.

Incidentally, the sum of the first $n$ triangular numbers is called the ***n*th tetrahedral number**.

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COEFFICIENTS OF BINOMIAL EXPANSIONS [#60-80]

The coefficient of a term is the number of times that term is in the sum of an expansion. For example, $(x - 3y)^2 = x^2 - 6xy + 9y^2$. The coefficient of the $x^2$ term is 1, the coefficient of the $xy$ term is $-6$, and the coefficient of the $y^2$ term is 9.

Luckily, there is an easy way to compute each of the coefficients.

When $(Ax+By)^n$ is expanded, each term is of the form $qx^ky^{n-k}$, where $q$ is the coefficient of the term. Notice that the sum of the powers of $x$ and $y$ add up to the original $n$. The coefficient $q$ is the product of three different parts: $q = (A^k)(B^{n-k})({}_n\mathrm{C}_k)$.

Recall that ${}_n\mathrm{C}_k = \dfrac{n!}{(n-k)! \cdot k!}$ and is called the combination "$n$ choose $k$." Here, $n!$ is called the **factorial** of $n$ and is equal to $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$.

For example, find the coefficient of the $x^2y^3$ term in the expansion of $(x+y)^5$. Solution: The coefficients of $x$ and $y$ in the binomial are both 1. Thus, the first two parts ($A^k$ and $B^{n-k}$) are both 1. We just need to find the combination part. The power $n$ is 5 and we need to use $k = 2$. This means we need ${}_5\mathrm{C}_2 = 10$. The coefficient $q = 1 \cdot 1 \cdot 10 = 10$.

Another example: find the coefficient of the $xy^3$ term in the expansion $(2x+3y)^4$. Solution: To find $q$, we have $(2^1)(3^3)({}_4\mathrm{C}_1) = 2(27)(4) = 216$.

Finally, one extra trick: the sum of all of the coefficients of a binomial expansion of $(Ax+By)^n$ is just $(A+B)^n$. As an example, the sum of the coefficients of the binomial expansion of $(2x+5y)^3$ is $7^3 = 343$.

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Practice Questions – The following practice questions cover the above examples and should be used to guide your inquiries into the new types of questions to be asked on the number sense tests.

1. $26 \times 13 =$ ________.
2. $81 \times 13 =$ ________.
3. $162 \times 13 =$ ________.
4. $311 \times 811 =$ ________.
5. $815 \times 715 =$ ________.
6. $915 \times 615 =$ ________.
7. The sum of the first 4 triangular numbers is ________.
8. The sum of the first 10 triangular numbers is ________.
9. The coefficient of the $x^2y^2$ term in the binomial expansion of $(x+y)^4$ is ________.
10. The coefficient of the $x^4y$ term in the binomial expansion of $(x-y)^5$ is ________.
11. The coefficient of the $xy$ term in the binomial expansion of $(3x+2y)^2$ is ________.
12. The coefficient of the $x^3y^2$ term in the binomial expansion of $(2x+5y)^5$ is ________.
13. The sum of the coefficients in the binomial expansion of $(x+y)^5$ is ________.
14. The sum of the coefficients in the binomial expansion of $(5x-2y)^4$ is ________.

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This document was prepared by Doug Ray for competition, 2013 and 2014. If you have any questions about the material presented, please email `doug@academicmeet.com`.